Relaxation function theory of dynamic spin susceptibility in layered copper oxides: Implications for neutron resonance peak and ω/T scaling

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Using the Mori-Zwanzig projection operator procedure, the relaxation function theory with a three pole approximation is presented for doped two-dimensional $S = 1/2$ Heisenberg antiferromagnetic (AF) system in the paramagnetic state. The role of AF short range order, its evolution with doping and temperature, is highlighted in view of the magnetic response of high-T_c layered cuprates spanning in frequency from neutron scattering down to magnetic resonance experiments. It is shown that the spin-wave-like theory is able to reproduce the main features of dynamic spin susceptibility in the high-T_c cuprates as observed experimentally.

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Magnetic properties of layered copper oxide High Temperature Superconductors (high-T_c's) remain the one of the intriguing topics of condensed matter.[1,2] The carrier free, parent to high-T_c, compounds are the two dimensional Heisenberg antiferromagnetic (2DHAF) insulators with short range order in which the temperature dependence of the magnetic correlation length ξ was described by a quantum nonlinear σ model[3,4] in accord with neutron scattering (NS) experiments[5] in carrier free La_2CuO_4 and has been studied then, e.g., by isotropic spin-wave theory.[6]

Among the most prominent features of the imaginary part of dynamic spin susceptibility $\chi''(\mathbf{k},\omega)$ revealed by NS studies is the observation of the resonance peak[7] at AF wave vector $\mathbf{Q} = (\pi,\pi)$ and frequency $\omega_r \approx 40$ meV in optimally doped $YBa_2Cu_3O_7$, and the ω/T scaling of the averaged over the Brillouin zone the imaginary part of dynamic spin susceptibility, $\chi''(\omega,T) = \int \chi''(\mathbf{q},\omega,T)\mathrm{d}^2\mathbf{q} \approx \chi''(\omega, T \to 0) f(\omega/T)$, in the underdoped high-T_c compounds.[1,8] The ω/T scaling of $\chi''(\omega,T)$ above T_c is referred to a nearby quantum critical point,[9] nevertheless, the theories[10,11] used the temperature dependence of ξ that *disagrees* with the experimental data.[8] Further studies[12] by means of nuclear magnetic/quadrupole resonance (NMR/NQR) revealed the extension of the universal behavior of $\chi''(\omega,T)$ down to MHz frequency range. For a decade the resonant feature was attributed only to the double layered cuprates, meanwhile, the observation[13] of the resonance peak in the single layered $Tl_2Ba_2CuO_{8+y}$ compound designate it as a generic feature of the layered copper oxides.

Most theories for resonance peak relate it with superconductivity (SC) because of the same energy scale or the resonance features are the feedback effect of SC that arises predominantly in the d-wave channel.[11,14–16] However, Fong *et al.*,[17] and most recently Stock *et al.*[18,19] have reemphasized the spin-wave like features of spin susceptibility in the underdoped $YBa_2Cu_3O_{6.5}$ above T_c, suggesting that the resonant features may be caused *not only* by the emergence of the SC state. In addition, Hwang, Timusk, and Gu[20] have recently shown by means of infrared spectroscopy that the resonance peak disappears completely in the overdoped $Bi_2Sr_2CaCu_2O_{8+y}$ sample with $T_c = 55$ K, thus suggesting the magnetic origin of the resonance peak. Note also the results of NS studies[21] of spin excitations in Zn/Ni impurity substituted $YBa_2Cu_3O_7$ above $T_c \approx 80$ K.

In this Communication, using the Mori-Zwanzig projection operator procedure with a *three* pole approximation for the relaxation function,[22–24] we will show that the resonant features in doped 2DHAF, as well as the nearly universal ω/T dependence of $\chi''(\omega,T)$ may be explained within a spin-wave-like theory, where the correlation length, its doping dependence and saturation with lowering temperature, governs the main features of $\chi''(\mathbf{k},\omega)$ as observed experimentally and *no* SC will be presupposed in the present calculations.

We employ the t-J Hamiltonian[25] since it is the minimal model to describe the electronic properties of high-T_c cuprates:

$$H_{t-J} = \sum_{i,j,\sigma} t_{ij} X_i^{\sigma 0} X_j^{0\sigma} + J\sum_{i>j}(\mathbf{S}_i\mathbf{S}_j - \frac{1}{4}n_i n_j), \qquad (1)$$

written in terms of the Hubbard operators $X_i^{\sigma 0}$ that create an electron with spin σ at site i and $\mathbf{S}_i$ are spin-1/2 operators. Here, the hopping integral $t_{ij} = t$ between the nearest neighbors (NN) describes the motion of electrons causing a change in their spins and $J = 0.12$ eV is the NN AF coupling constant. The spin and density operators are defined as follows: $S_i^\sigma = X_i^{\sigma\tilde\sigma}$, $S_i^z = \frac{1}{2}\sum_\sigma \sigma X_i^{\sigma\sigma}$, $n_i = \sum_\sigma X_i^{\sigma\sigma}$, $(\sigma = -\tilde\sigma)$, with the standard normalization $X_i^{00} + X_i^{++} + X_i^{--} = 1$.

We formulate our study of the spin fluctuations following Mori,[22] who showed it's efficiency for both the classical (and essential equivalence to Brownian motion) and quantum (e.g. Heisenberg systems of arbitrary dimension) many body systems.[23] The time evolution of a dynamical variable $S_{\mathbf{k}}^z(\tau)$, say, is given by

$$\dot{S}_{\mathbf{k}}^z(\tau) \equiv \frac{dS_{\mathbf{k}}^z(\tau)}{d\tau} = iLS_{\mathbf{k}}^z(\tau). \qquad (2)$$

The Liouville superoperator L corresponds to the commutator with the Hamiltonian (1): $iLS_{\mathbf{k}}^z(\tau) \to [H, S_{\mathbf{k}}^z(\tau)]$. The projection of the vector $S_{\mathbf{k}}^z(\tau)$ onto the $S_{\mathbf{k}}^z \equiv S_{\mathbf{k}}^z(\tau = 0)$ axis is given by: $\mathcal{P}_0 S_{\mathbf{k}}^z(\tau) = R(\mathbf{k},\tau)\cdot S_{\mathbf{k}}^z$, and defines the linear projection Hermitian operator $\mathcal{P}_0$. One may separate $S_{\mathbf{k}}^z(\tau)$ into the projective and vertical components with respect to the $S_{\mathbf{k}}^z$ axis: $S_{\mathbf{k}}^z(\tau) = R(\mathbf{k},\tau)\cdot S_{\mathbf{k}}^z + (1-\mathcal{P}_0)S_{\mathbf{k}}^z(\tau)$, where $R(\mathbf{k},\tau) \equiv (S_{\mathbf{k}}^z(\tau),(S_{-\mathbf{k}}^z)^*)\cdot(S_{\mathbf{k}}^z,(S_{-\mathbf{k}}^z)^*)^{-1}$ is the relaxation function in the inner-product bracket notation: $(S_{\mathbf{k}}^z(\tau),(S_{-\mathbf{k}}^z)^*) \equiv k_BT\int_0^{1/k_BT} d\varrho \langle e^{\varrho H} S_{\mathbf{k}}^z(\tau) e^{-\varrho H}(S_{-\mathbf{k}}^z)^*\rangle$, and the angular brackets denote the thermal average.

For future evaluations, it is convenient to introduce a set of quantities $f_0(\tau)$, $f_1(\tau)$, ..., $f_j(\tau)$, ... defined by equations: $f_j(\tau) \equiv \exp(iL_j\tau)f_j \equiv \exp(iL_j\tau)iL_jf_{j-1}$, where $L_0 \equiv L$, $f_0(\tau) \equiv S^z_{\mathbf{k}}(\tau)$, $L_j \equiv (1-\mathcal{P}_{j-1})L_{j-1}$, and $\Delta_j^2 \equiv (f_j, f_j^*) \cdot (f_{j-1}, f_{j-1}^*)^{-1}$ for $j \geq 1$. The set $\{f_j\}$ forms an orthogonal set. The *larger* number of f_j is used, the *finer* description of $S^z_{\mathbf{k}}(\tau)$ is obtained. The last quantity from this set f_n, affected by evolution operator $\exp(iL_n\tau)$, resulting in $f_n(\tau)$, is called the "n-th order random force",[22] acting on the variable $S^z_{\mathbf{k}}(\tau)$ and is responsible for fluctuation from its average motion.

In terms of Laplace transform of the relaxation function, $R(\mathbf{k},\tau)$, one may construct a continued fraction representation for $R^L(\mathbf{k},s)$, for which Lovesey and Meserve[26,23] suggested a three pole approximation, $R^L(\mathbf{k},s) = \int_0^\infty d\tau\, e^{-s\tau}R(\mathbf{k},\tau) \approx 1/\{s+\Delta^2_{1\mathbf{k}}/[s+\Delta^2_{2\mathbf{k}}/(s+1/\tau_{\mathbf{k}})]\}$, with a cutoff characteristic time $\tau_{\mathbf{k}} = \sqrt{2/(\pi\Delta^2_{2\mathbf{k}})}$, by arguing that $S^z_{\mathbf{k}}(\tau)$ fluctuations are weakly affected by the higher order random forces. For the relaxation shape function $F(\mathbf{k},\omega) = Re[R^L(\mathbf{k},i\omega)]/\pi$, this gives

$$F(\mathbf{k},\omega) = \frac{\tau_{\mathbf{k}}\Delta^2_{1\mathbf{k}}\Delta^2_{2\mathbf{k}}/\pi}{[\omega\tau_{\mathbf{k}}(\omega^2-\Delta^2_{1\mathbf{k}}-\Delta^2_{2\mathbf{k}})]^2+(\omega^2-\Delta^2_{1\mathbf{k}})^2}, \quad (3)$$

where $\Delta^2_{1\mathbf{k}}$ and $\Delta^2_{2\mathbf{k}}$ are related to the frequency moments

$$\langle\omega^n_{\mathbf{k}}\rangle = \int_{-\infty}^{\infty} d\omega\; \omega^n F(\mathbf{k},\omega) = \frac{1}{i^n}\left[\frac{d^nR(\mathbf{k},\tau)}{d\tau^n}\right]_{\tau=0}, \quad (4)$$

of $R(\mathbf{k},\tau)$ as $\Delta^2_{1\mathbf{k}}=\langle\omega^2_{\mathbf{k}}\rangle$, $\Delta^2_{2\mathbf{k}}=(\langle\omega^4_{\mathbf{k}}\rangle / \langle\omega^2_{\mathbf{k}}\rangle)-\langle\omega^2_{\mathbf{k}}\rangle$ for $\tau \gtrsim \tau_{\mathbf{k}}$. Note that $F(\mathbf{k},\omega)$ is real, even in both $\mathbf{k}$ and ω, normalized to unity $\int_{-\infty}^{\infty} d\omega F(\mathbf{k},\omega)=1$, and is related to $\chi''(\mathbf{k},\omega)$ as

$$\chi''(\mathbf{k},\omega) = \omega\chi(\mathbf{k})F(\mathbf{k},\omega). \quad (5)$$

The expression for the second moment is straightforward, $\langle\omega^2_{\mathbf{k}}\rangle = i\langle[\dot{S}^z_{\mathbf{k}}, S^z_{-\mathbf{k}}]\rangle/\chi(\mathbf{k}) = -\left(8Jc_1 - 4t_{eff}T_1\right)(1-\gamma_{\mathbf{k}})/\chi(\mathbf{k})$, while it is rather cumbersome for $\langle\omega^4_{\mathbf{k}}\rangle = i\langle[\ddot{S}^z_{\mathbf{k}}, \dot{S}^z_{-\mathbf{k}}]\rangle/\chi(\mathbf{k})$ and is not reproduced here (see Ref. 27 for details). Note that in the expression for $\langle\omega^4_{\mathbf{k}}\rangle$ the decoupling procedures were employed for the thermodynamic averages in spirit of papers by Hubbard and Jain,[28] and by Kondo and Yamaji.[29] The averages with four operators are approximated, as usually, by products of two-operator correlation functions,[26] however, multiplied now with the decoupling parameter ζ, e.g.: $\langle S^\sigma_i S^{\bar\sigma}_r S^\sigma_m S^{\bar\sigma}_j\rangle \to \zeta\langle S^\sigma_i S^{\bar\sigma}_r\rangle\langle S^\sigma_m S^{\bar\sigma}_j\rangle$ and so on. This parameter may be fixed from the total moment sum rule, however, the uncertainty in the correlation length and the destruction of fraction of the Cu^{2+} moments by holes makes this restriction less rigorous and we fix ζ from the comparison with NS data.

TABLE I. The calculated NN AF spin-spin correlation function $c_1 = \frac{1}{z}\sum_\rho\langle S^z_i S^z_{i+\rho}\rangle$, the parameter g_-, and the spin stiffness constant ρ_S using the expressions and the procedure as described in Refs. 30, 31 in the $T \to 0$ limit together with the n_ξ and ζ values as extracted from comparison with NS data.

δ	c_1	g_-	$2\pi\rho_S/J$	n_ξ	ζ
0.04	−0.1055	3.913	0.30	2	1.0
0.09	−0.0851	3.46	0.24	∼1.5	2.8
0.14	−0.0657	3.034	0.15	1	4.0

In the present work we employ the static quantities from Ref. 30, that has been *derived* for the 2DHAF systems doped by charge carriers and work in the overall temperature range. The expression for static spin susceptibility is given by,[30]

$$\chi(\mathbf{k}) = \frac{4|c_1|}{Jg_-(g_++\gamma_{\mathbf{k}})}, \quad (6)$$

and its structure is the same as in the isotropic spin-wave theory.[6] The meaning of g_+ is clear: it is related to ξ via the expression $\xi/a = 1/(2\sqrt{g_+-1})$, where a =3.8 Åis a lattice unit. The transfer amplitude between the NN is given by the spectral theorem: $T_1 \equiv -\frac{1}{z}\sum_\rho\langle X^{\sigma 0}_i X^{0\sigma}_{i+\rho}\rangle = p\sum_{\mathbf{k}}\gamma_{\mathbf{k}} f^h_{\mathbf{k}}$, the index ρ runs over NN, $f^h_k = [\exp(-E_{\mathbf{k}}+\mu)/k_BT+1]^{-1}$ is the Fermi function of holes, $\gamma_{\mathbf{k}} = \frac{1}{z}\sum_\rho\exp(i\mathbf{k}\rho) = \frac{1}{2}(\cos k_xa + \cos k_ya)$, and $z=4$. The number of *extra* holes, due to doping, δ, per one plane Cu^{2+}, can be identified with the Sr content x in $La_{2-x}Sr_xCuO_4$. The chemical potential μ is related to δ by $\delta = p\sum_{\mathbf{k}} f^h_{\mathbf{k}}$, where $p=(1+\delta)/2$. The excitation spectrum of holes is given by $E_{\mathbf{k}} = 4t_{eff}\gamma_{\mathbf{k}}$, where the hoppings, t, are affected by electronic and AF spin-spin correlations c_1, resulting in *effective* values,[25,31,32] for which we set $t_{eff} = \delta J/0.2$ to match the insulator-metal transition.

In the present calculations we will use the doping and temperature dependence of ξ following the explicit formulation given in Ref. 30. To mimic the low temperature behavior of the correlation length we use the expression, as in Ref. 31, resulting in *effective* correlation length ξ_{eff}, given by,

$$\xi^{-1}_{eff} = \xi_0^{-1} + \xi^{-1}. \quad (7)$$

Here, in Eq. (7), ξ is affected by doped holes, in contrast with the Keimer *et al.*[8] empirical equation, where ξ is given by the Hasenfratz-Niedermayer formula[4] and there was no influence of the hole subsystem on ξ. For strongly doped systems the expression for ξ is much more complicated compared with simple relation $\xi/a \simeq \frac{J\sqrt{g_-}}{k_BT}\exp(2\pi\rho_S/k_BT)$, which is valid for carrier free or lightly doped systems.[30,27] Thus from now on we replace ξ by ξ_{eff}. In the best fit of ξ_{eff} to experimental data[8,9] (see Fig. 5a below) we use $\xi_0 = a/n_\xi\delta$, where n_ξ is given in Table I. Whether its value is related to stripe ordering[33] or more exotic states[34] remains to be shown.

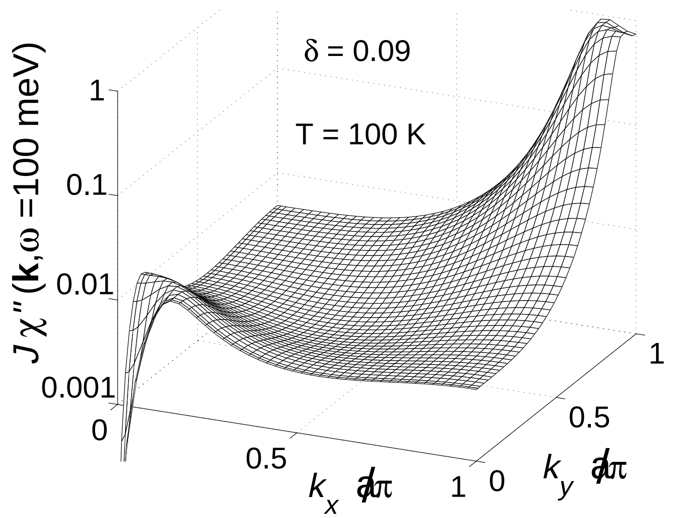

FIG. 1. Semilog-scale mesh of the calculated imaginary part of dynamic spin susceptibility $\chi''(\mathbf{k},\omega)$ in the Brillouin zone.

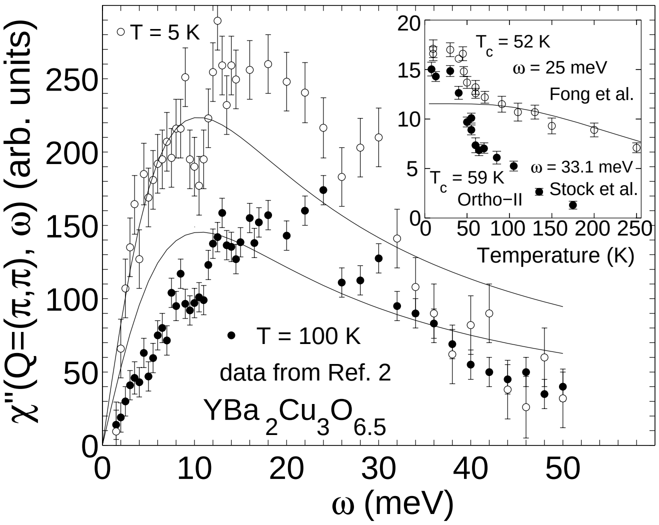

FIG. 2. Imaginary part of the odd spin susceptibility $\chi''(\mathbf{Q},\omega)$ as obtained from NS studies[2,17,18] of $YBa_2Cu_3O_{y\approx6.5}$ samples *versus* frequency ω. The lower solid line shows the calculated $\chi''(\mathbf{Q},\omega)$ for $T = 100$ K and the upper solid line for $T = 5$ K scaled up by a factor of 1.5. The inset shows $\chi''(\mathbf{Q},\omega)$ *versus* T in arb. units for each data set. The solid line shows the calculated temperature dependence of $\chi''(\mathbf{Q}=(\pi,\pi),\omega = 10$ meV$)$ scaled to fit the data above T_c.

The results of the calculations are summarized in Table I. For brevity we consider here the three cases: $La_{2-x}Sr_xCuO_4$ with $x = 0.04$ and $x = 0.14$, and $YBa_2Cu_3O_{6.5}$ for which we accept[18] $\delta = 0.09$, and, particularly, $p = (1 + \delta/2)/2$, due to the bilayered structure that affects also $\langle\omega^2_{\mathbf{k}}\rangle$, $\langle\omega^4_{\mathbf{k}}\rangle$, and ξ.

Figures 1-5 show the wave vector, frequency, doping and temperature dependence of $\chi''(\mathbf{k},\omega)$. First, we note that for all temperatures $F(\mathbf{q},\omega)$ gives the elastic peak at $q = 0$ and $\omega = 0$. Fig. 1 shows that the diffusive (small $\mathbf{k}$) dynamics is negligible, the calculated $\chi''(\mathbf{k},\omega)$ for $\delta = 0.09$ is peaked at $\mathbf{Q} = (\pi,\pi)$ for $\omega < 55$ meV and becomes incommensurate with a spin-wave like cone (symmetric ring of scattering) for $\omega \gtrsim 55$ meV in agreement with the high-energy NS studies of Stock *et al.*[19] Fig. 2 shows $\chi''(\mathbf{Q},\omega)$ *versus* frequency and temperature. The inset in Fig. 2 shows that $\chi''(\mathbf{Q},\omega)$ may not exhibit the sharp increase below T_c, in contrast with the predictions within the weak coupling theories.[16] Indeed, it seems, that the more underdoped $YBa_2Cu_3O_{y\approx6.5}$ sample (controlled by T_c) with the smaller resonance frequency shows the smaller increase of $\chi''(\mathbf{Q},\omega)$ below T_c.

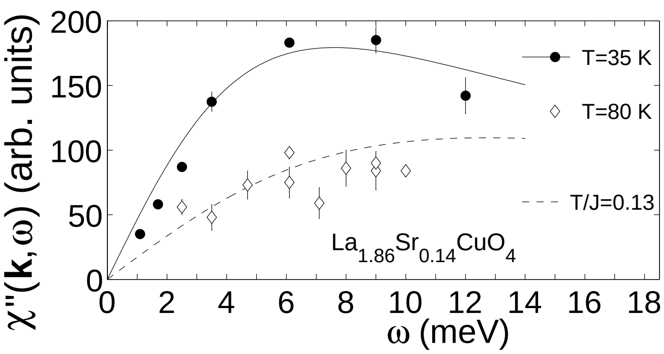

FIG. 3. The imaginary part of dynamic spin susceptibility $\chi''(\mathbf{k},\omega)$ *versus* ω (symbols: NS data for $La_{1.86}Sr_{0.14}CuO_4$ of the incommensurate peak from Ref. 9 and the lines show the calculated $\chi''(\mathbf{Q}=(\pi,\pi),\omega)$).

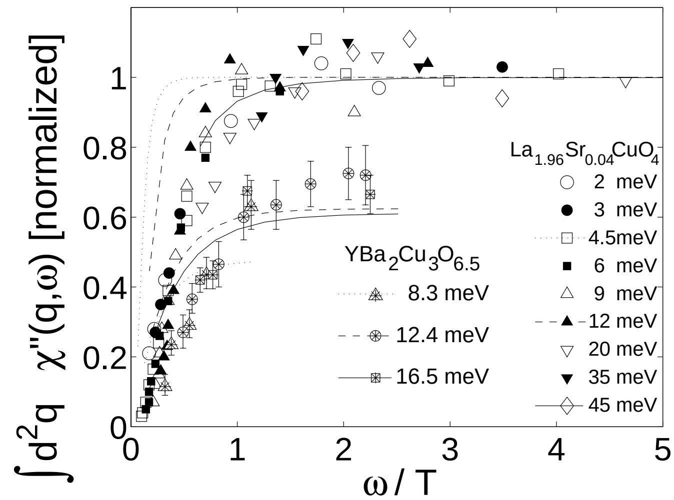

FIG. 4. The averaged over the Brillouin zone imaginary part of dynamic spin susceptibility $\chi''(\omega,T) = \int\chi''(\mathbf{q},\omega,T)d^2\mathbf{q}$ *versus* ω/T (symbols: NS data for $La_{1.96}Sr_{0.04}CuO_4$ from Ref. 8 and for $YBa_2Cu_3O_{6.5}$ from Ref. 18, the lines show the calculated $\chi''(\omega,T)$). For $YBa_2Cu_3O_{6.5}$ the scale of the calculated $\chi''(\omega,T)$ has been fitted to the NS data.[18]

The results of our calculations in spirit of undamped spin-wave picture of Kondo and Yamaji,[29] presented in Fig. 3, show that the damping of spin-wave like excitations affects $\chi''(\mathbf{k},\omega)$ noticeably in doped 2DHAF even at low temperatures. Noting that the relaxation shape function $F(\mathbf{k},\omega)$ can be understood within the spin-wave like[23] framework, $\omega^{SW}_{\mathbf{k}}{=}2\int_0^\infty d\omega\,\omega F(\mathbf{k},\omega)$, the temperature and doping dependence of the damping of the spin-wave-like excitations may be studied further. Our results suggest that, in contrast with Ref. 11, the damping of spin-wave like excitations is, however, does not qualitatively affects $\chi''(\mathbf{k},\omega)$ even in the normal state of optimally doped high-T_c cuprates. Note also the oversimplifications in Ref. 11 in the expression for susceptibility, the use of the temperature independent correlation length parameter, as indeed observed[8] *only* at $T < 400$ K in the *lightly doped* regime, and the numerical results that are valid in the $T \gtrsim J/2 \approx 700$ K limit.

Fig. 4 shows the averaged over the Brillouin zone and normalized imaginary part of dynamic spin susceptibility $\chi''(\omega,T)$ *versus* ω/T and suggests the ω/T scaling for underdoped high-T_c layered cuprates with a deviations in qualitative agreement with NS data[8] and we compare the results of our calculations with NMR data that allows to check also the absolute values of $\chi''(\mathbf{k},\omega)$ with considerably smaller ω.

The nuclear spin-lattice relaxation rate $1/T_1$ is given by

$$^{63}(1/T_1) = \frac{2k_BT}{\omega_0}\sum_{\mathbf{k}}{}^{63}F(\mathbf{k})^2\chi''(\mathbf{k},\omega_0), \qquad (8)$$

where $\omega_0{=}2\pi{\cdot}35$ MHz $\simeq1.45{\cdot}10^{-4}$ meV ($\ll T, J$) is the measuring NQR frequency. The quantization axis of the electric field gradient coincides with the crystal axis c which is perpendicular to CuO_2 planes defined by a and b. The wave vector dependent hyperfine formfactor for plane ^{63}Cu sites[36,37] is given by, $^{63}F(\mathbf{k})^2{=}(A_{ab} + 4\gamma_{\mathbf{k}}B)^2$, where $A_{ab}{=}1.7{\cdot}10^{-7}$ eV and $B{=}(1 + 4\delta)\cdot 3.8\cdot 10^{-7}$ eV are the Cu on-site and transferred hyperfine couplings, respectively. The relation for B is used to match the weak changes with Sr doping.[39]

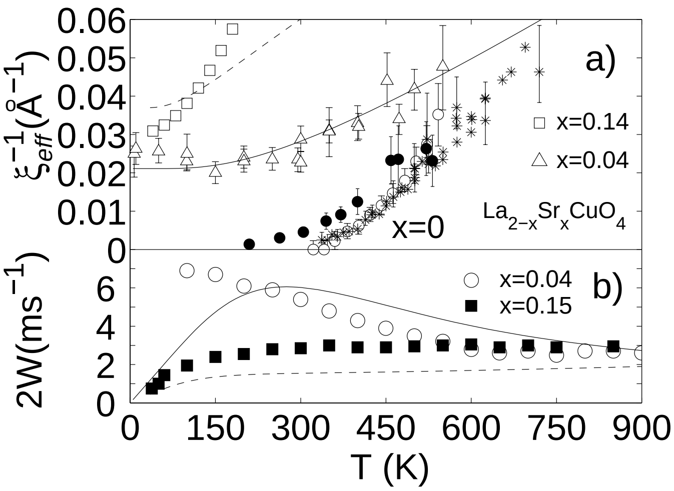

FIG. 5. The calculated temperature dependencies for $x = 0.04$ (solid lines) and for $x = 0.14$ (dashed lines) of (a) inverse correlation length ξ_{eff}^{-1}, fitted to the NS data ($x = 0.04$ from Ref. 8 and for $x = 0.14$, modeled by $\xi_{x=0.14}^{-1} = \sqrt{k_o^2 + a^{-2}(T/E)^{2/Z_A}}$, with $k_o = 0.03$ Å^{-1}, $E = 690$ K, and $Z_A = 0.8$, following Ref. 9), (the data for $x = 0$ shown by circles and asterisks from Refs. 5, 8, and 35), and (b) plane copper nuclear spin-lattice relaxation rate $^{63}(1/T_1) = 2W$ from Ref. 12. The deviation of the calculations from the experimental data for $x \approx 0.14$ may be caused by the effect of doped holes on the exchange coupling constant J.

Fig. 5 shows the temperature dependencies of inverse correlation length and plane copper nuclear spin-lattice relaxation rate, $^{63}(1/T_1)$. Eqs. (5,8), and Fig. 5 show that the temperature dependence of $^{63}(1/T_1)$ is governed by the temperature dependence of the correlation length and by the factor k_BT. At low T, where $\xi_{eff} \simeq const$, the plane copper $^{63}(1/T_1) \propto T$, as it should. At high T, the correlation length shows the weak doping dependence and behaves similarly to that of carrier free La_2CuO_4 and $^{63}(1/T_1)$ of doped samples behaves similarly to that of La_2CuO_4. Thus our results shown in Fig. 5 suggest that the "pseudogap" effect seen with NMR in the high-T_c cuprates is *hidden* in the correlation length that affects the observable quantities, and, generally, is in agreement with the conclusion based on the nearly AF Fermi liquid concept[38,39] about the leading role of the correlation length in temperature and doping dependence of $1/T_1$.

In conclusion, it is shown that the weak ω/T dependence of $\chi''(\mathbf{k}, \omega)$ as well as the resonant feature may be explained within the undamped spin-wave-like theory and the resonant feature seen by NS may be not the only feedback effect of emergence of the superconducting state. The influence of AF short range order (affected by stripes or, possibly, more exotic phases) on the *dynamic* quantities is discussed, possessing a reasonable agreement with the observations by means of neutron scattering and magnetic resonance experiments in the underdoped high-T_c layered copper oxides. At the same time, the future model for magnetic properties should match also a crossover from the localized spins picture (magnon-like excitation) to itinerant weak coupling theory with doping.

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